Transformações Geométricas

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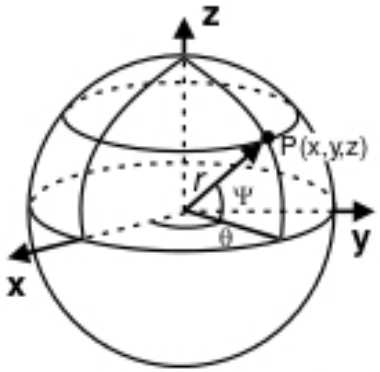
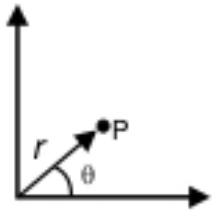
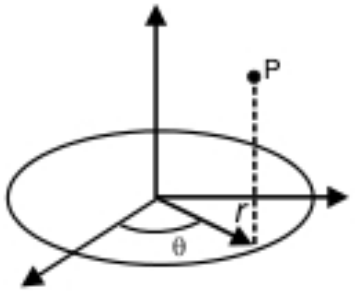
Sistemas de Coordenadas



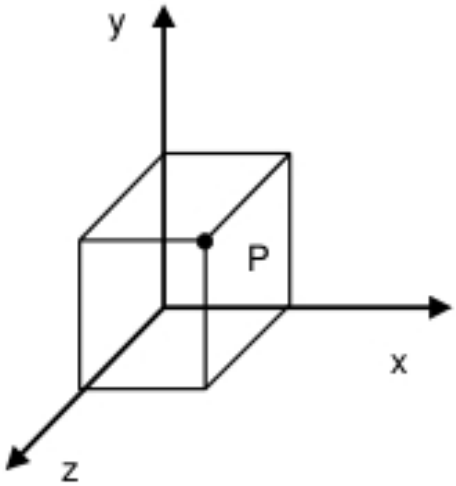
Coordenadas Esféricas



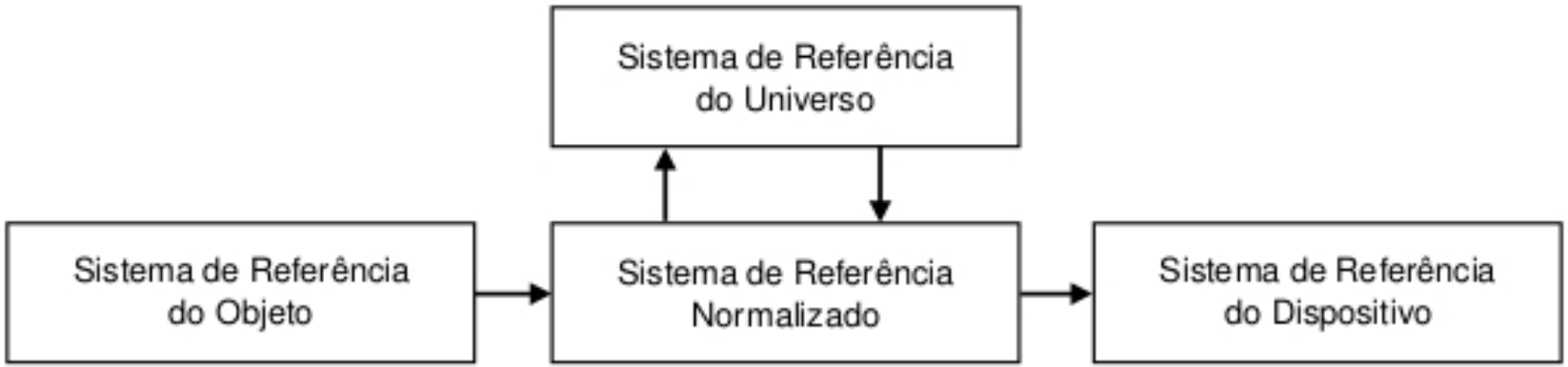
Coordenadas Polares



Coordenadas Cilíndricas

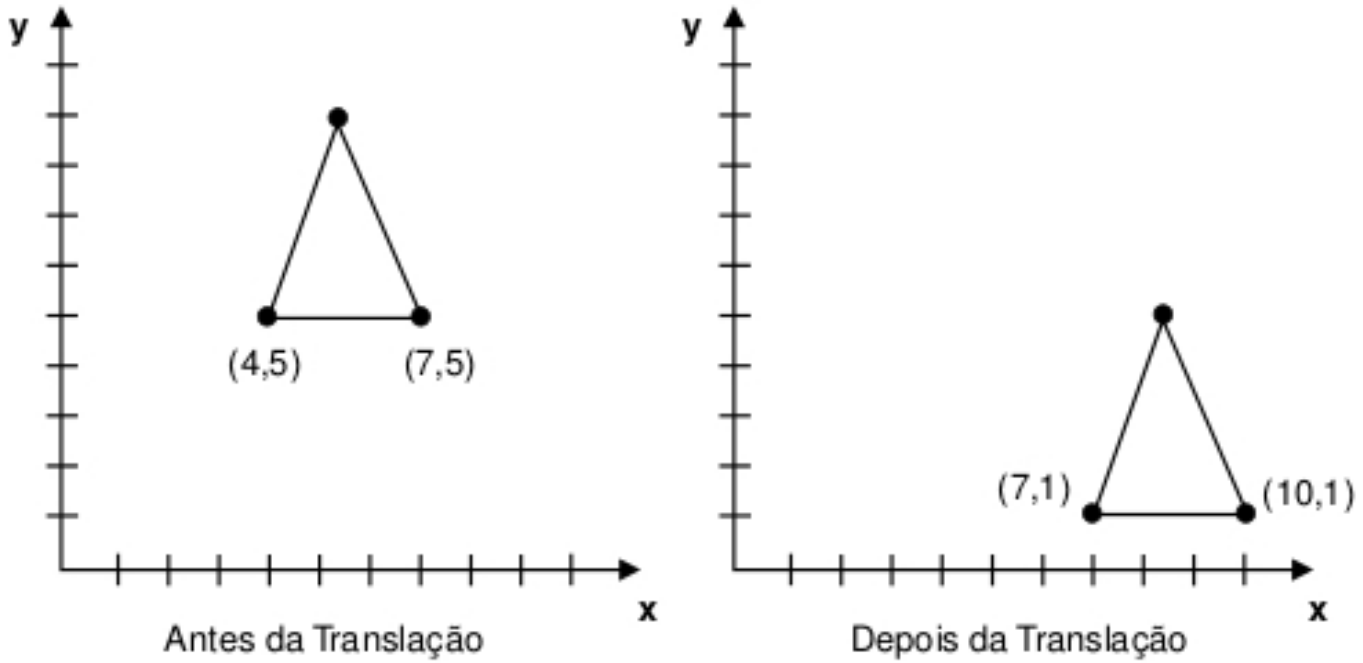


Sistemas de Referência



Transformação de Translação

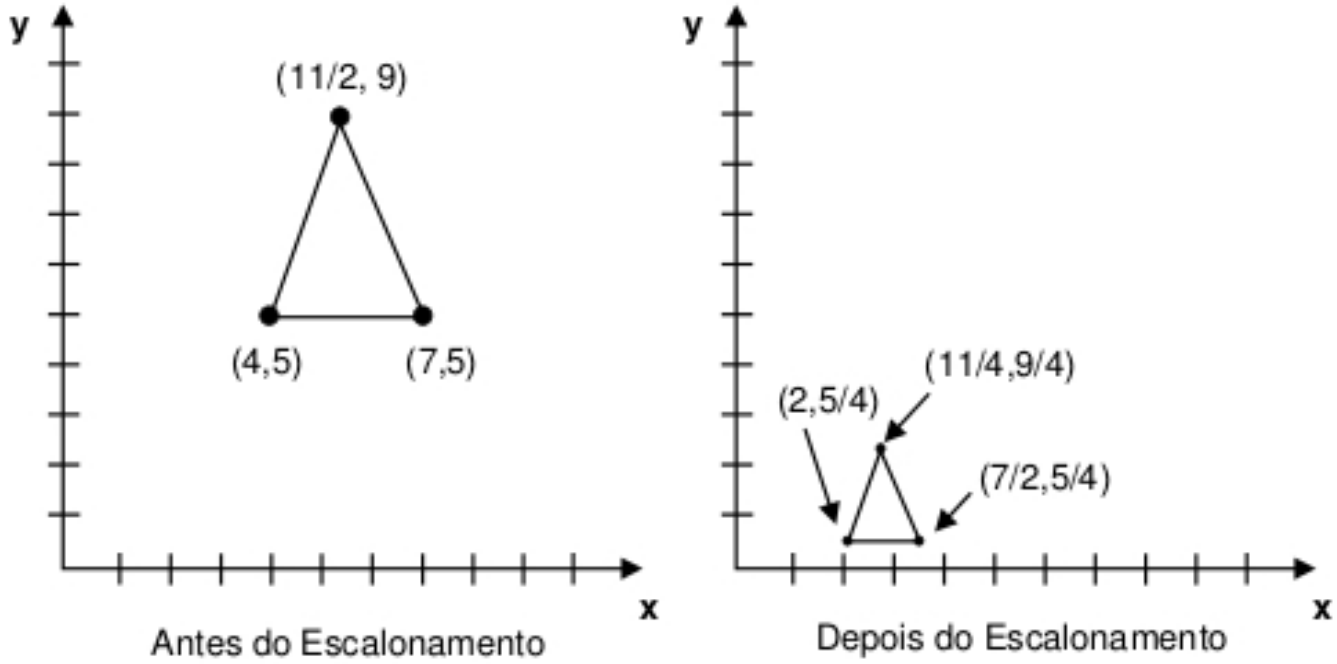
$$x' = x + T_x$$
$$y' = y + T_y$$



$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} + \begin{bmatrix} T_x & T_y & T_z \end{bmatrix}$$

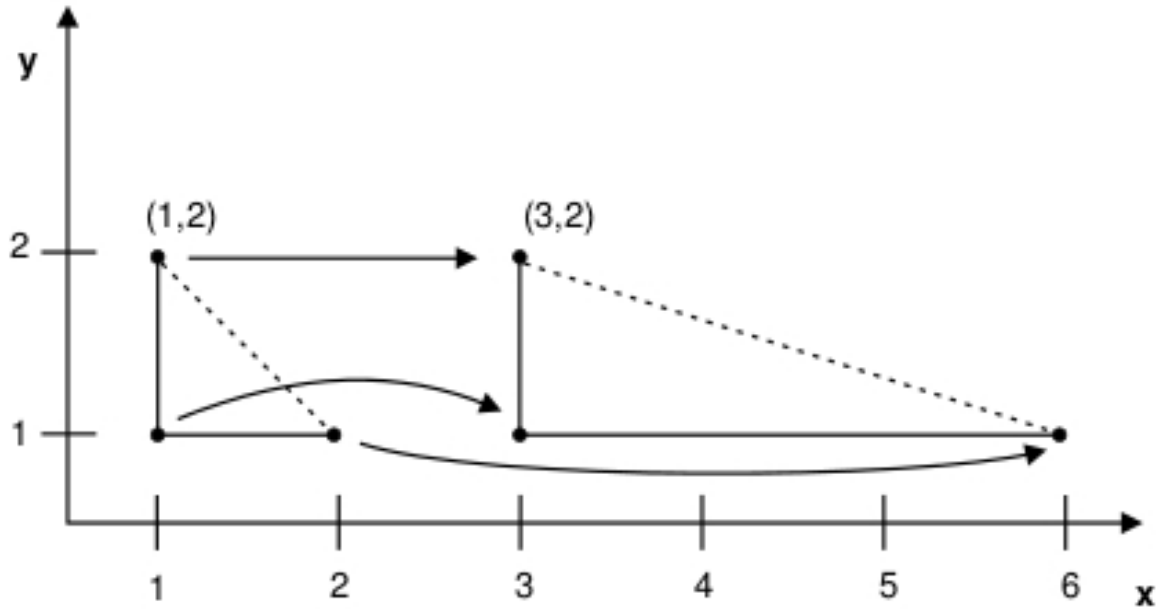
Transformação de Escala

$$x' = x \bullet S_x$$
$$y' = y \bullet S_y$$



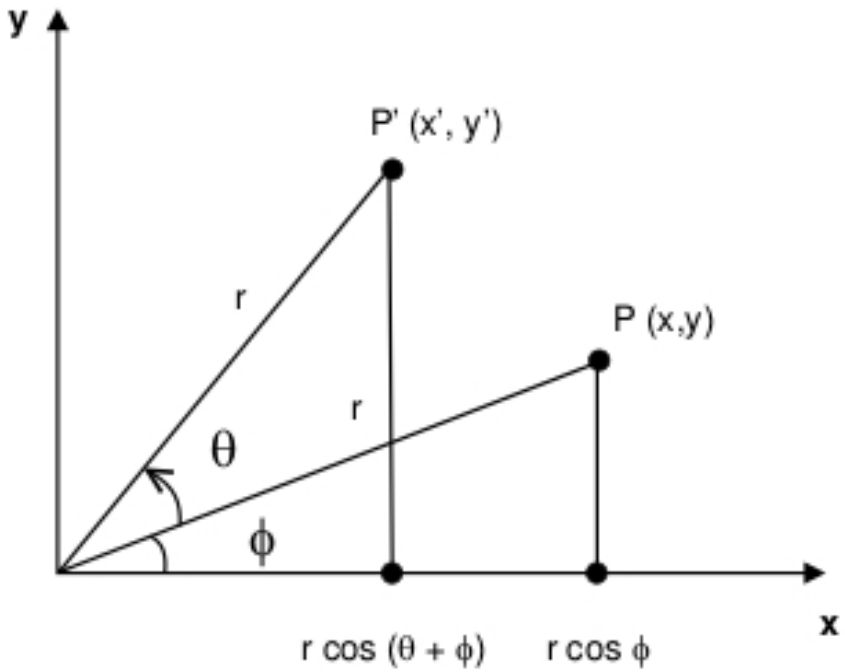
$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} = \begin{bmatrix} xS_x & yS_y & zS_z \end{bmatrix}$$

Transformação de Escala



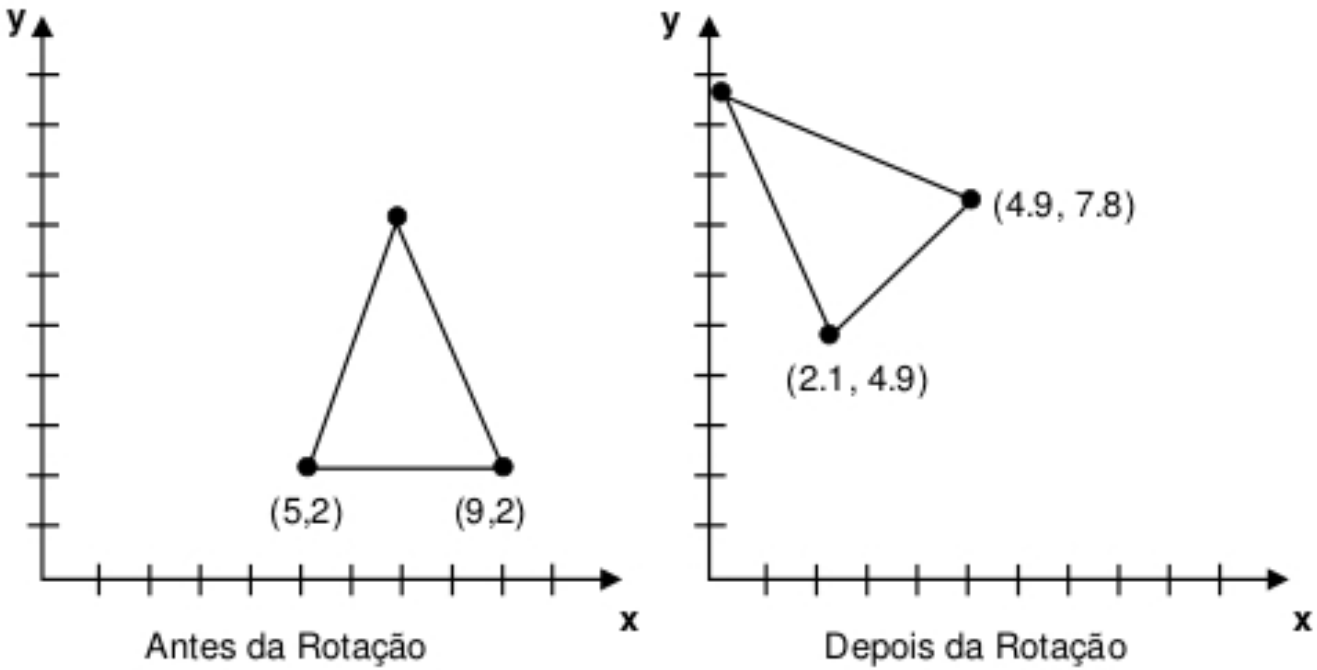
$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} xS_x & yS_y \end{bmatrix}$$

Transformação de Rotação

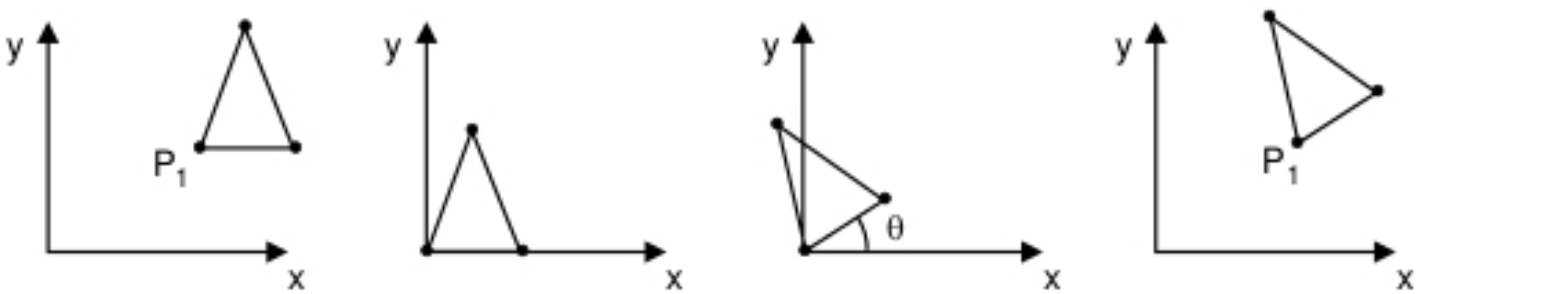


$$x' = r.\cos(\theta + \phi) = r.\cos\phi.\cos\theta - r.\sin\phi.\sin\theta$$
$$y' = r.\sin(\theta + \phi) = r.\sin\phi.\cos\theta + r.\cos\phi.\sin\theta$$
$$x' = x.\cos\theta - y.\sin\theta$$
$$y' = y.\cos\theta + x.\sin\theta$$
$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Transformação de Rotação



Transformação de Rotação



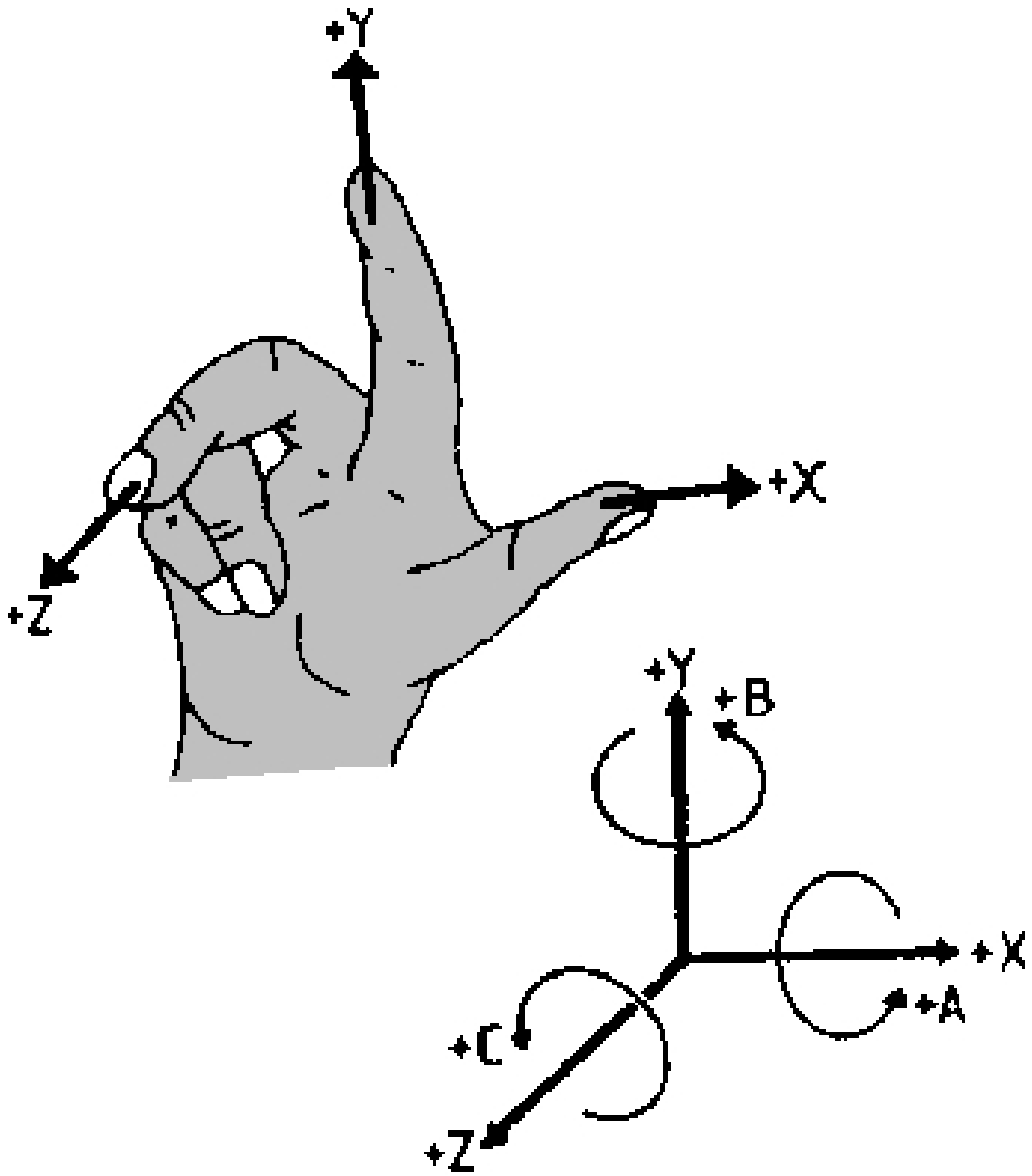
Objeto Original

Depois da Translação de P_1 à origem

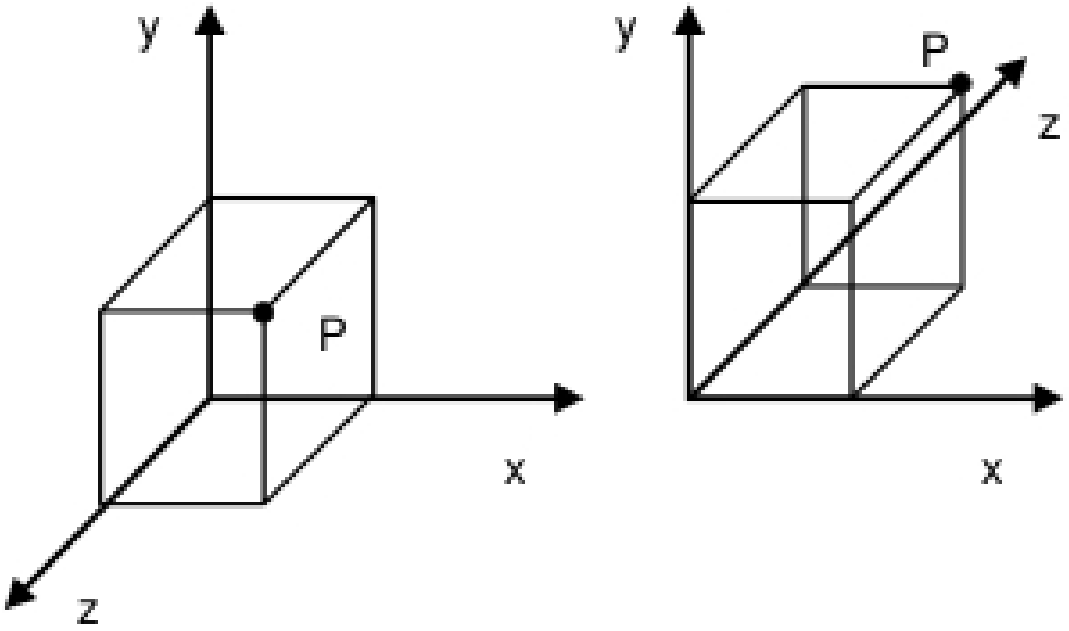
Após Rotação

Após Translação que retorna a posição original

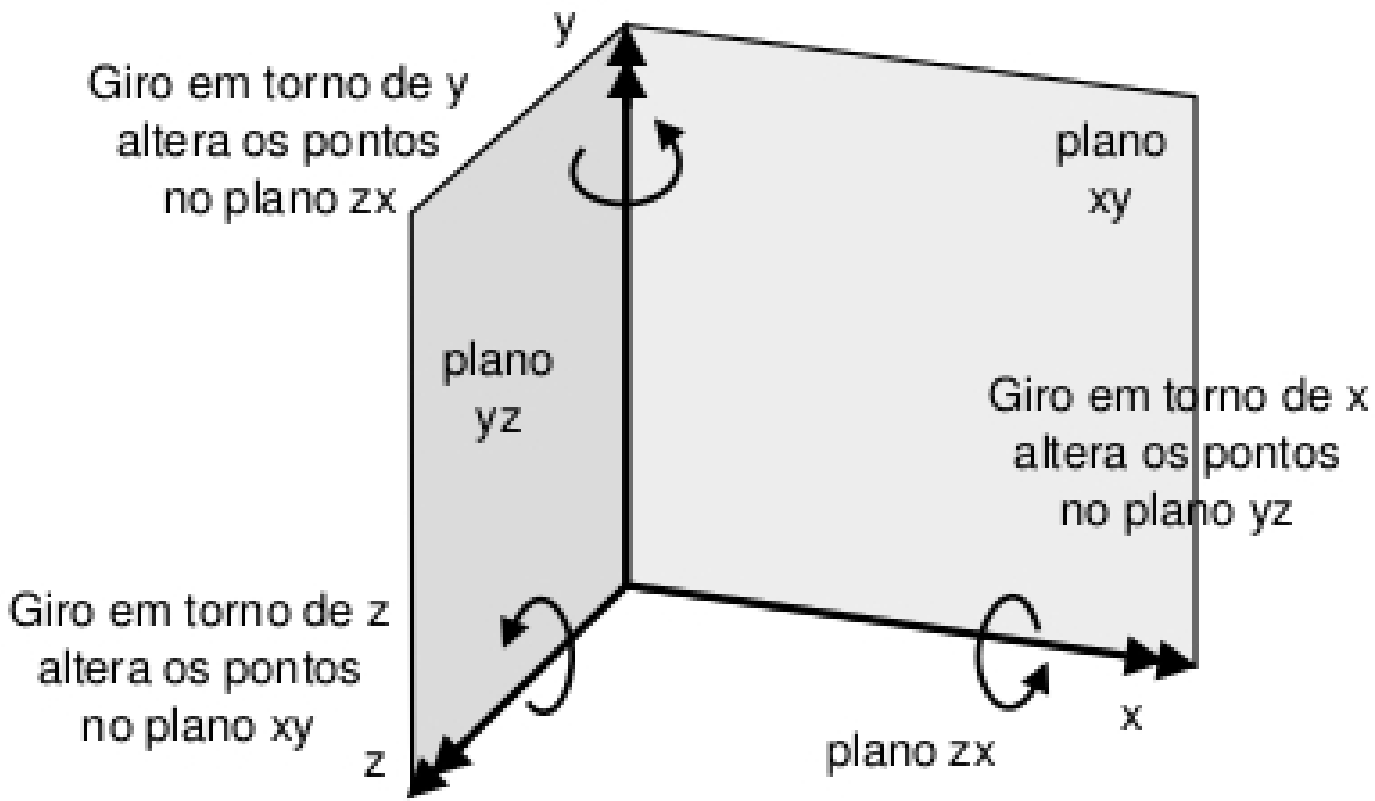
Regra da mão direita



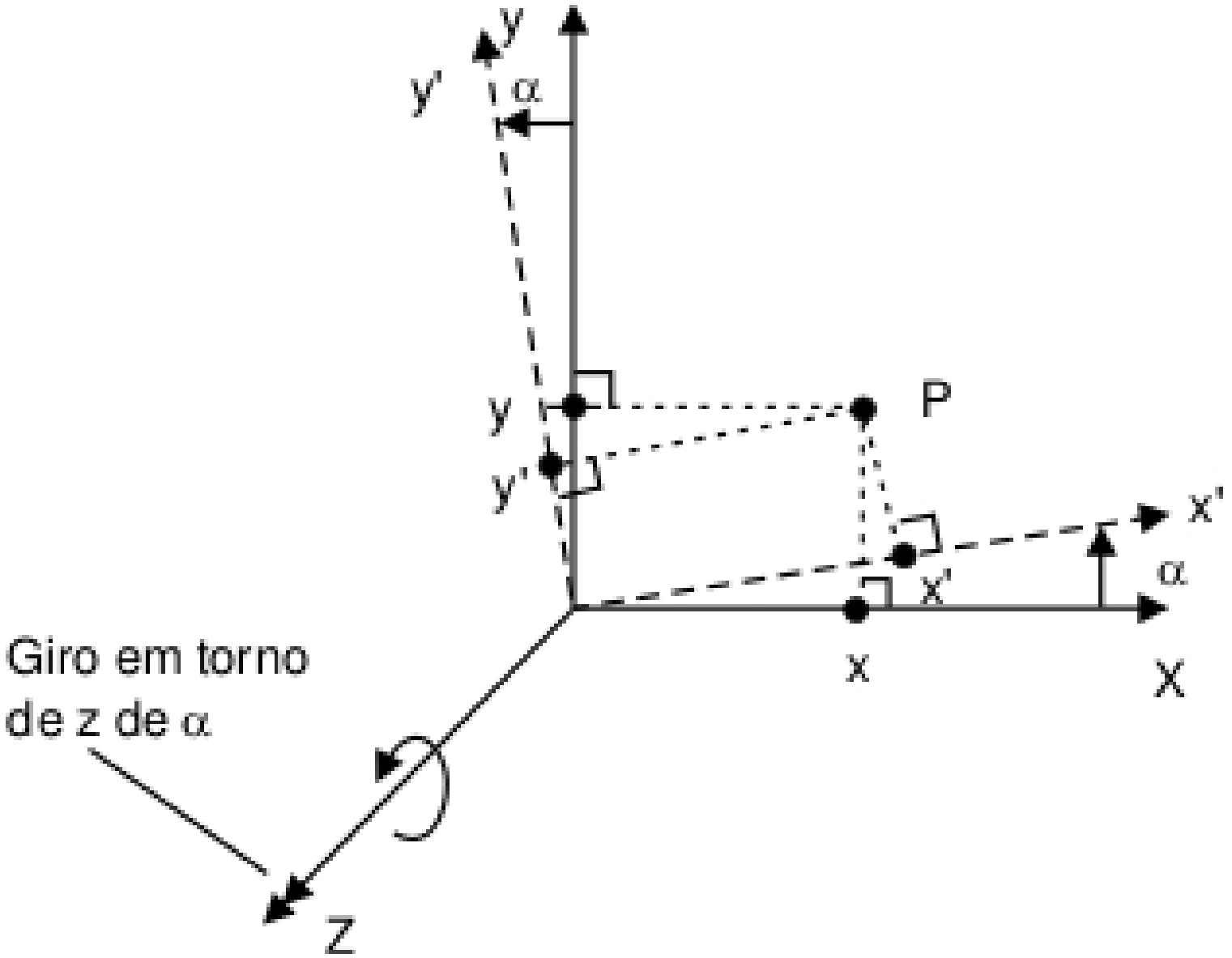
Regra da mão direita



Ângulos de Euler



Ângulos de Euler



Transformação de Rotação

$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix}$$

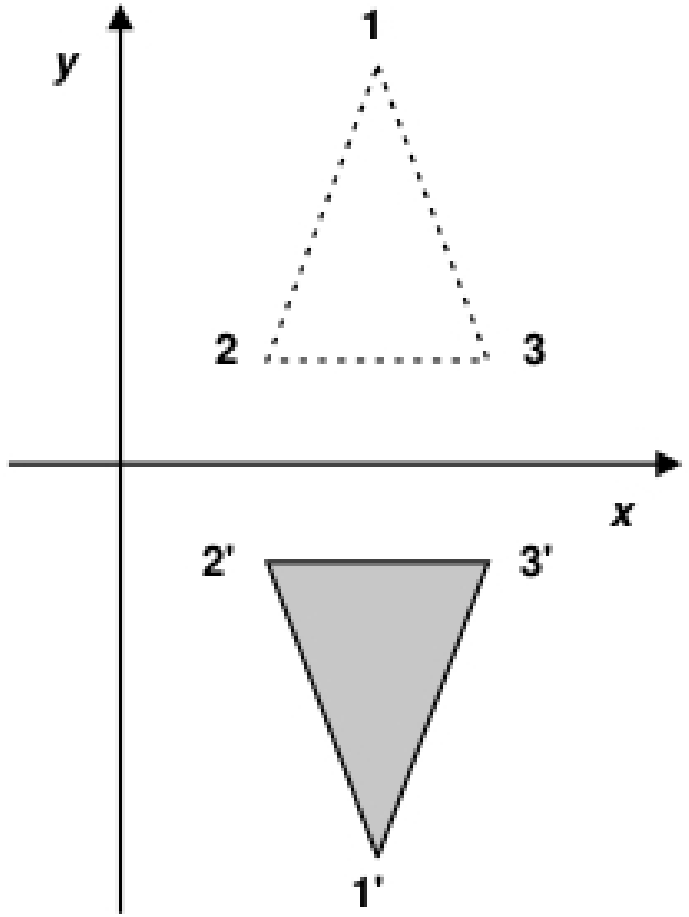
$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} \cos\sigma & 0 & -\sin\sigma \\ 0 & 1 & 0 \\ \sin\sigma & 0 & \cos\sigma \end{bmatrix}$$

Transformação de Rotação

$$\begin{bmatrix} x' & y' & z' \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 10^\circ & \sin 10^\circ \\ 0 & -\sin 10^\circ & \cos 10^\circ \end{bmatrix}$$

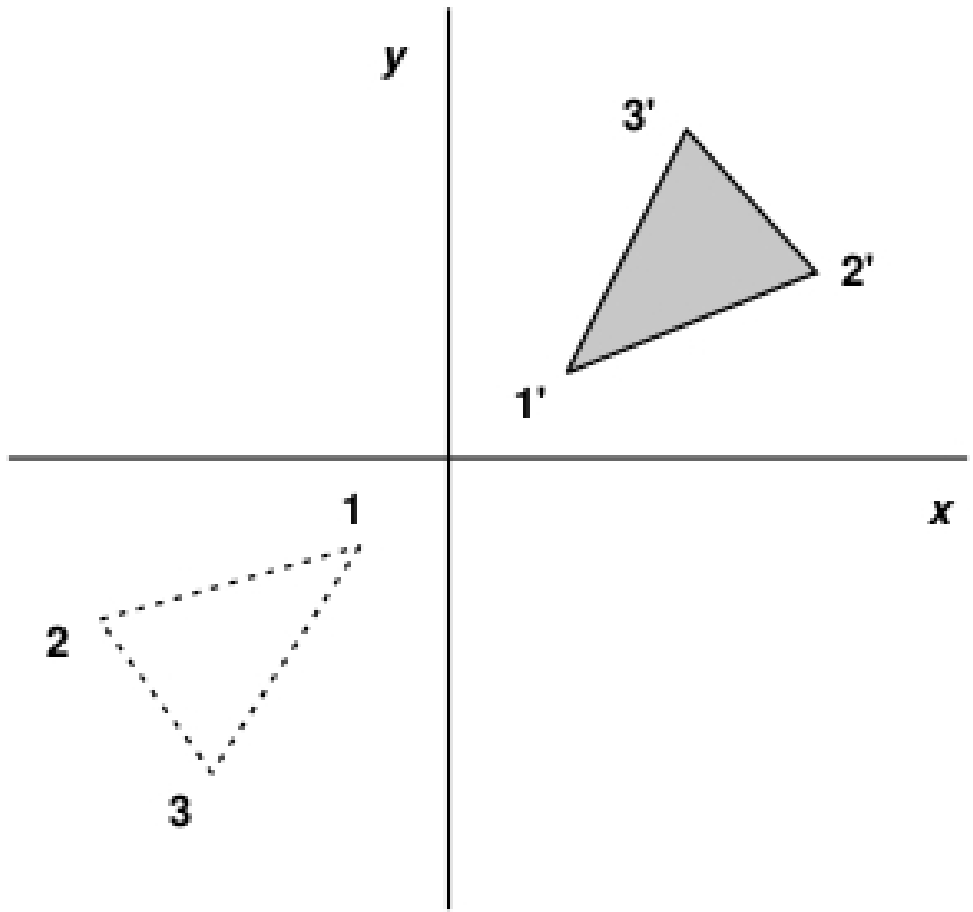
$$\begin{bmatrix} \cos 20^\circ & 0 & -\sin 20^\circ \\ 0 & 1 & 0 \\ \sin 20^\circ & 0 & \cos 20^\circ \end{bmatrix} \begin{bmatrix} \cos 30^\circ & \sin 30^\circ & 0 \\ -\sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformação de Reflexão



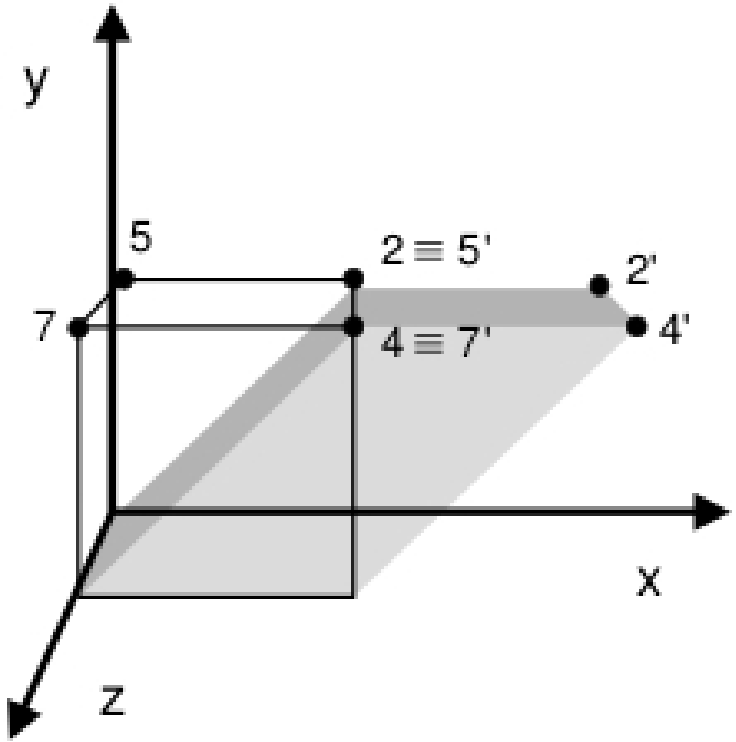
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformação de Reflexão



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

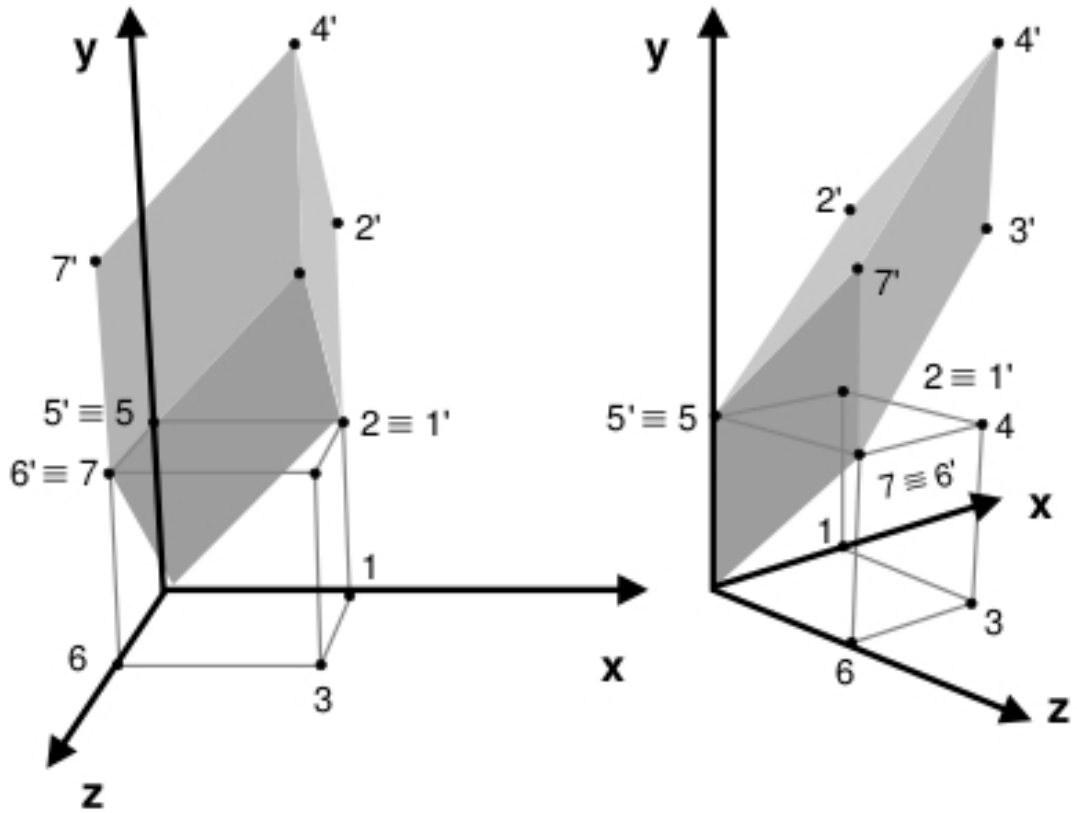
Transformação de Cisalhamento



$$\begin{bmatrix} 1 & 0 & 0 \\ S & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x + Sy$, $y' = y$ e $z' = z$

Transformação de Cisalhamento



$$\begin{bmatrix} 1 & a & 0 \\ S & 1 & 0 \\ 0 & b & 1 \end{bmatrix}$$

Coordenadas Homogêneas

$$P = \begin{bmatrix} x' & y' & z' & M \end{bmatrix}$$

$$\begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} \frac{x'}{M} & \frac{y'}{M} & \frac{z'}{M} \end{bmatrix}$$

$$\begin{bmatrix} x' & y' & z' & 1 \end{bmatrix} = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

FIM